DESIGN OF L-NETWORKS FOR MATCHING ANTENNAS TO TRANSMITTERS

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G3HGM was first licensed in 1947, as D2HN, while serving in Royal Signals in the British Zone of occupied Germany. After demobilization he returned to his native Scotland and obtained the callsign GM3HGM. He worked for the North British Locomotive Company in Glasgow until 1954 when he moved south of the border, changing his callsign to G3HGM. Since then he has worked in engineering with, first, Vauxhall Motors Ltd at Luton; and then in the National Health Service. He hopes that imminent early retirement will provide more time for operating, as opposed to construction and measurement which have been his predominant interests.

Introduction

The article "Measurement of antenna radiation resistance and reactance" by John Bazley, G3HCT, *Rad Com* June 1979, presented information on the construction of a most useful device for measuring complex impedances, slanted towards antenna systems. Unfortunately, it is my experience that most recently-licensed amateurs (and especially black-box operators, who are apparently willing and able to pay three-digit sums for commercial antenna matching units) and a goodly number of old-timers, would experience some difficulty in telling a complex impedance apart from a bull's foot. This article has been prepared in an attempt to help experimenters who "just cannot get my antenna to load up om", and especially to show how to translate into inductors and capacitors the information which can be obtained from G3HCT's device.

Basic principles

At the outset, it might be useful to explain simply one or two of the properties possessed by the components we employ in atus and transmitter tank circuits. These are resistors, capacitors and inductors.

(1) All three have resistance, denoted R. This is independent of frequency.

(2) In addition, capacitors have capacitive reactance which decreases with rising frequency and vice versa, and has the effect of causing the current flowing through a capacitor to lead in phase by 90° the voltage across it. Capacitive reactance is denoted as X_C and has a magnitude

$$\frac{1}{2\pi fC}$$

(3) In addition, inductors have inductive reactance which increases with rising frequency, and vice versa, and has the effect of causing the current flowing through an inductor to lag in phase by 90° the voltage across it. Inductive reactance is denoted by X_L and has a magnitude $2\pi fL$.

The combined effect of R and X_C in a capacitor, and R and X_L in an inductor, is known as impedance, and cannot be arrived at by simply adding the resistance and the reactance. Instead, a process known as vectorial addition is used, since allowance must be made for the fact that the currents in the reactances are out of phase with the voltages, and also with the currents in any resistance present. Vectorial addition is very simply done by means of a vector diagram, and the following examples will show how:

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(a) Find the impedance of an inductor whose resistance is 3Ω and reactance at a given frequency is 4Ω (see Fig 1).

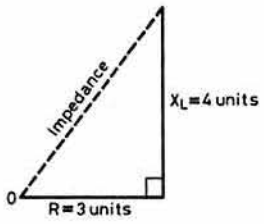

Fig 1. Vector diagram showing how the impedance of a combination of resistance (three units) and inductive reactance (four units) is arrived at. The vector representing inductive reactance is conventionally regarded as positive, and is therefore drawn upwards from the end of the resistance vector. In the resulting right-angled triangle, (impedance)$^2 = R^2 + X^2_L$, so impedance = $\sqrt{R^2 + X^2_L}$

Starting at 0, draw a horizontal line three units in length to represent the 3Ω resistance of the inductor. From its end draw a line vertically upwards and four units long to represent the inductive reactance. If the end of this line is now joined to 0, the length of the third line, to the same scale, will represent the impedance. A slight acquaintance with Pythagoras will tell one that, in this case, the answer is

$$\sqrt{3^2 + 4^2} = 5\Omega$$

(b) Find the impedance of a capacitor whose resistance is 1Ω and whose reactance at a given frequency is 3Ω (see Fig 2).

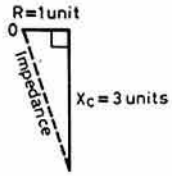

Fig 2. Vector diagram similar to Fig 1, but with the vector X_C drawn downwards, representing capacitive reactance. Impedance = $\sqrt{R^2 + X^2_C}$

The method is the same as in *(a)* except that the vector representing X_C is drawn downwards. This is mathematical convention, X_C being regarded as a negative vector, and X_L as positive. The numerical value of the impedance is, of course, unaffected by the direction of the vector representing the reactance, and in this case is

$$\sqrt{1^2 + 3^2} = \sqrt{10} = 3 \cdot 162\Omega$$

It must always be kept in mind that reactances, and the impedances of which they form part, are dependent on frequency. If the reader has seen the light so far, this may be a convenient point to look at the interesting special case of a circuit containing all three of the properties inductance, capacitance and resistance. This is shown in Fig 3.

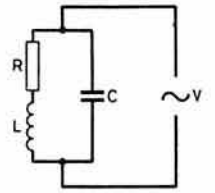

Fig 3. Tank circuit containing L, C and R. R is undesirable but is unavoidably present in the conductors of the coil and capacitor

Since X_C falls with rising frequency and vice versa, and X_L rises with rising frequency and vice versa, if an alternating voltage V is applied to the LC circuit shown, and then varied in frequency, it must be possible to find a setting at which $X_L = X_C$ and since X_L is positive and X_C is negative, they are mutually cancelling. This is the phenomenon known as resonance, and in this condition only the resistance present (in the wire of the inductor, the

connecting leads and the capacitor) limits the size of the current which flows. So next time you are tuning your pa tank for maximum dip, remember that what you are really doing is making $X_L = X_C$, and that the coil and connections are made of the thickest wire possible (maybe even silver-plated) to ensure that R is low and, as a result, current is maximized.

Once a little facility has been developed in thinking in the above terms, it will be possible to appreciate that very few antennas are likely to be purely resistive; ie possess no capacitive or inductive reactance. After all, the wire has inductance and distributed capacitance, plus capacitance to earth etc. This seems a pity, since transmitter power amplifiers are designed to work into a specific resistive load, and coaxial and other feeder cables also should ideally be terminated with a load having an impedance equal to the characteristic impedance of the cable. Fig 4 shows a fairly typical arrangement at the output end of a transmitter. From this, it can be seen that the job of the atu is to produce a situation where the combination of everything within the dotted enclosure (atu, long wire and earth system) presents 75Ω to the incoming coaxial cable from the transmitter.

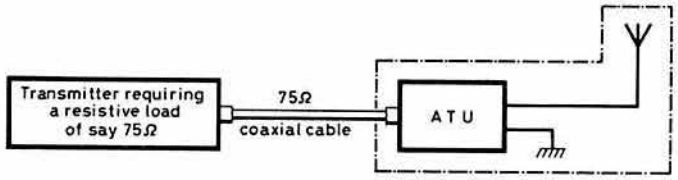

Fig 4. The atu is adjusted in such a way that the combination of atu + antenna + earth is resistive at the operating frequency, and equal to the characteristic impedance of the feeder

The problem, therefore, resolves itself into one of designing an atu capable of transforming the complex impedance (ie involving L, C and R) of an antenna/earth system into a resistive load to suit the transmitter output characteristics. Practically all design information for atus makes the quite unwarranted assumption that the antenna/earth system impedance is resistive. The following information will, it is hoped, show how to design L-networks capable of dealing with reactive loads, ie practically all antenna/earth systems.

Design considerations

One must, of course, start by measuring the resistive and reactive components of the radiating system, using the G3HCT device, at the frequency at which it is to be used. It should be noted that, if the installation is one in which a long wire antenna is to be worked against earth, the proposed permanent earth must be in use when the readings are taken.

L-networks come in two configurations (Fig 5). Note that the parallel element is at the high impedance end in both cases. Suppose that the parameters of your antenna/earth system have been measured, and that it has a resistance of 22Ω and a capacitive reactance of 7Ω. Mathematical shorthand is used to express this complex impedance as 22-j7. Had the reactance been inductive, of 7Ω, the impedance would have been written 22 + j7. The design of a network to match an antenna/earth system impedance of 22 − j7 to a transmitter requiring a 75Ω resistive load at 3600kHz is carried out as follows:

Table 1. Coil winding details

Inductance (μH)	Diameter (in)	Length (in)	SWG	TPI
2·2	0·75	2	18	8
2·9	0·75	2	18	10
3·05	1·0	3	18	6
5·1	1·0	3	18	8
6·85	0·75	2	20	16
7·1	1·25	5	16	6
7·5	1·0	3	18	10
9·6	1·5	5	16	6
11·9	1·75	5	14	6
12·25	1·25	5	16	8
14·0	0·75	2	22	24
15·25	2·0	5	14	6
16·75	1·5	5	16	8
18·25	1·25	5	16	10
21·12	1·75	5	14	8
23·1	2·5	5	14	6
25·25	1·5	5	16	10
27·12	2·0	5	14	8
28·0	0·75	2	24	32
33·7	1·75	5	16	10
41·0	2·5	5	14	8
42·3	2·0	5	16	10
45·5	1·25	5	20	16
62·25	1·5	5	20	16
64·0	2·5	5	16	10
72·0	2·0	3	24	32
84·48	1·75	5	18	16
108·47	2·0	5	18	16
164·0	2·5	5	18	16
185·0	1·25	5	24	32

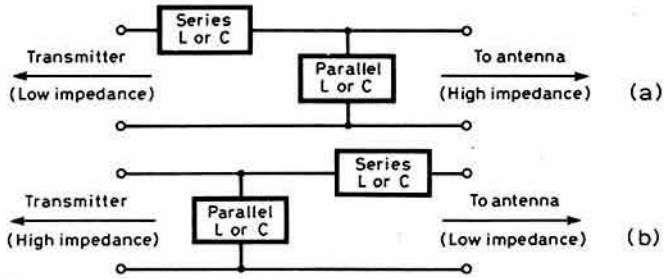

Fig 5. L-network layouts for matching: *(a)* an antenna system whose impedance is higher than the transmitter output impedance; *(b)* an antenna system whose impedance is lower than the transmitter output impedance

Case 1 (Resistive part of antenna/earth system less than desired transmitter load.) The use of the configuration in Fig 5(b) is required, and the network will look like Fig 6.

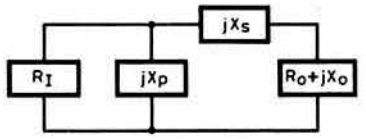

Fig 6. "Block" arrangement of elements if the resistive part of the antenna/earth system is less than the desired transmitter load

R_I = Transmitter load = network input resistance = 75Ω
R_o = Resistive part of antenna/earth system impedance = 22Ω
jX_o = Reactive part of antenna/earth system impedance = $-j7\Omega$

(1) $A = \sqrt{\frac{R_I - R_o}{R_o}} = \sqrt{\frac{75-22}{22}} = \sqrt{\frac{53}{22}} = \sqrt{2{\cdot}409} = 1{\cdot}552\Omega$

(2) $jX_s = -jX_o + jR_oA$
$= j7 + j(22)\,(1{\cdot}552) = j41{\cdot}146$

(3) Since jX_s is positive, the necessary reactance must be inductive.
The inductance required to give this reactance (41·146Ω) at 3,600kHz

$$L_s = \frac{X_s}{2\pi f} = \frac{41{\cdot}146}{2\times3{\cdot}14\times3{\cdot}6\times10^6}\text{henries} = 1{\cdot}819\times10^{-6}\text{henries} = 1{\cdot}819\mu\text{H}.$$

(4) $jX_p - j\frac{R_I}{A} = -j\frac{75}{1{\cdot}552} = -j48{\cdot}324$

(5) Since jX_p is negative, the necessary reactance must be capacitive.
The capacitance required to give this reactance (48·324Ω) at 3,600kHz is

$$C_P = \frac{1}{2\pi fXp} = \frac{1}{2\times3{\cdot}14\times3{\cdot}6\times10^6\times48{\cdot}324}\text{farads} =$$

$9{\cdot}15\times10^{-10}$ farads = 914pF.

This network would therefore take the form shown in Fig. 7.

In fact, alternative values of the reactances can be found by repeating the calculation, using the expression $jX_S = -jX_0 - jR_oA$ instead of that given at step **(2)**, and the expression $jX_p = j\frac{R_I}{A}$ instead of that given at step **(4)**.

This results in the following:
$jX_s = -j27{\cdot}144$ which is capacitive and requires 1,628 pF;
$jX_p = j48{\cdot}324$ which is inductive and requires 2·136 μH.
This network would be arranged as in Fig 8.

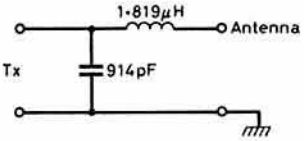

Fig 7. Circuit of Fig 6, with values of L and C required to match an antenna impedance of 22 − j7 to a transmitter output impedance of 75Ω, at 3,600kHz

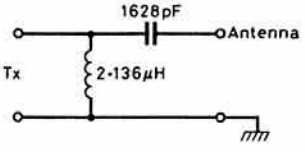

Fig 8. Alternative arrangement of reactances to give the same matching ability as Fig 7

Table 2. Summary of formulas used to calculate jX_s and jX_p

Resistive component of antenna/earth system less than required transmitter load	Resistive component of antenna/earth system greater than required transmitter load
$jX_s = -jX_o + jR_oA$	$jX_s = j\sqrt{Z_o^2\left(\frac{R_I}{R_o}\right) - R_I^2}$
and $jX_p = -j\frac{R_I}{A}$	and $jX_p = \frac{Z_o^2}{\left(\frac{R_o}{R_I}\right)\left(jX_s\right) + jX_o}$
OR	OR
$jX_s = -jX_o - jR_oA$	$jX_s = -j\sqrt{Z_o^2\left(\frac{R_I}{R_o}\right) - R_I^2}$
and $jX_p = j\frac{R_I}{A}$	and $jX_p = \frac{Z_o^2}{\left(\frac{R_o}{R_I}\right)\left(jX_s\right) - jX_o}$

If the antenna/earth system had had a resistance of, say, 92Ω and an inductive reactance of 12Ω, ie its impedance was 92 + j12, and it was desired to match this to a transmitter requiring a 75Ω resistive load at 3,600kHz, it would be necessary to proceed as follows:

Case 2 (Resistive part of antenna/earth system greater than desired transmitter load). The use of the configuration in Fig 5(a) is required, and the network will look like Fig 9.

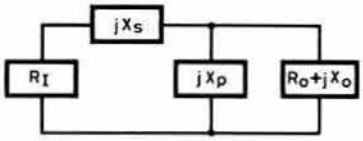

Fig 9. "Block" arrangement of elements if the resistive part of the antenna/earth system is greater than the desired transmitter load

R_I = transmitter load = network input resistance = 75Ω.
R_o = resistive part of antenna/earth system impedance = 92Ω.
jX_o = reactive part of antenna/earth system impedance = +j12Ω.

(1) $Z_o^2 = R_o^2 + X_o^2 = 92^2 + 12^2 = 8464 + 144 = 8,608$

(2) $jX_s = j\sqrt{Z_o^2\left(\frac{R_I}{R_o}\right) - R_I^2}$

$= j\sqrt{8608\left(\frac{75}{92}\right) - 75^2}$

$= j\sqrt{7017 - 5625} = j\sqrt{1392} = j37{\cdot}3$

(3) $L_s = \frac{X_s}{2\pi f} = \frac{37{\cdot}3}{2\times 3{\cdot}14\times 3{\cdot}6\times 10^6}$ henries $= 1{\cdot}65\times 10^{-6}$ henries $= 1{\cdot}65\mu$H.

(4) $jX_p = \frac{Z_o^2}{\left(\frac{R_o}{R_I}\right)\left(jX_s\right) + jX_o} = \frac{8,608}{\frac{92}{75}\times j37{\cdot}3 + j12}$

$= \frac{8,608}{j45{\cdot}75 + j12} = \frac{8,608}{j57{\cdot}75}$

$= \frac{-j8,608}{57{\cdot}75}$ (multiplying top and bottom by $-j$) $= -j149$

(5) $C_P = \frac{1}{2\pi f X_P} = \frac{1}{2\times 3{\cdot}14\times 3{\cdot}6\times 149\times 10^6}$ farads $=$
$2{\cdot}97\times 10^{-10}$ farads $= 297$pF.

This network would take the form shown in Fig 10.

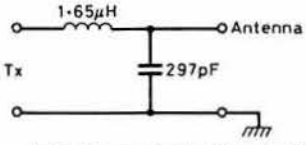

Fig 10. Circuit of Fig 9, with values of L and C required to match an antenna impedance of 92 + j12 to a transmitter output impedance of 75Ω, at 3,600kHz

As in the example using configuration 5(b), there are alternative values of the reactances which will give the same practical results, and these can be arrived at by using the expression

$$jX_s = -j\sqrt{Z_o^2\left(\frac{R_I}{R_o}\right) - R_I^2}$$

instead of that given at step (2), and the expression

$$jX_p = \frac{Z_o^2}{\left(\frac{R_o}{R_I}\right)\left(jX_s\right) + jX_o}$$

instead of that given at step (4).
This results in the following:
$jXs = -j37{\cdot}3$ which is capacitive and requires 1,186pF
$jXp = j149{\cdot}05$ which is inductive and requires $6{\cdot}59\mu$H.
This network would be arranged as in Fig 11.

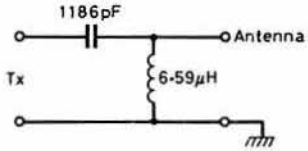

Fig 11. Alternative arrangement of reactances to give the same matching ability as Fig 10

Note that, upon occasion, both series and parallel elements may be capacitors. It is equally possible for both to be inductors.

Construction

The necessary design information is now to hand to enable an L-network to be constructed to solve your problems. The only remaining difficulty is the translation of the inductance values into actual coils. Table 1 gives details of 30 coils and, where the desired value does not appear, interpolation will give results of adequate accuracy.

If it is required to construct a multiband antenna matching unit, the characteristics of the antenna/earth system will need to be examined for each band, and the calculations carried out to determine the maximum and minimum limits required, in inductance and capacitance. The elements will require to be made variable. In the case of capacitors this may be done by incorporating switched fixed capacitors in parallel with a single variable capacitor. The fixed capacitors should preferably be air spaced, but, failing this, moulded mica is suitable for hf use. Inductors may be varied by switched taps, roller-coasters, variometers, or insertion and withdrawal of a suitable core.

In order to eliminate totally the tendency to work blind, the set-up of Fig 12 is recommended.

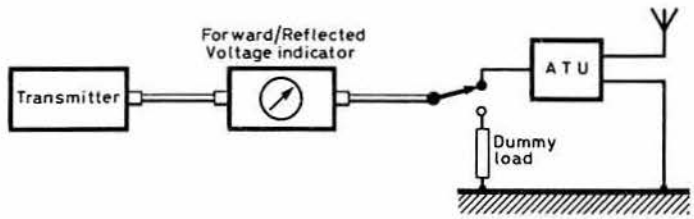

Fig 12. Convenient practical arrangement to permit fast and accurate tune-up

Setting up

The correct setting-up procedure is as follows:

(1) Connect dummy load of correct resistance in place of the atu/antenna combination.
(2) Adjust transmitter output loading and tuning until rated current is drawn at maximum dip.
(3) Adjust sensitivity of forward/reflected voltage indicator for full scale reading in the forward direction. Check the reflected voltage, which should be zero.
(4) Do not alter transmitter adjustments, but switch from dummy load to atu and adjust its controls until the pa anode current is the same as in step (2) and reflected voltage reading is zero.

The combination of the atu and the antenna/earth system is now resistive and of the correct magnitude. You are properly tuned up!

Acknowledgements

1. "L-Networks for Reactive Loads", Robert E. Gordon, W0KFI. *QST* September 1966.
2. Descriptive literature on "CODAR-QOILS". □

Table 3. Formulas used to calculate C_s, C_p, L_s and L_p once X_s and X_p are known

$C_s = \frac{1}{2\pi f X_s}$	$L_s = \frac{X_s}{2\pi f}$
$C_p = \frac{1}{2\pi f X_p}$	$L_p = \frac{X_p}{2\pi f}$

Units are: C in farads; L in henries; f in hertz; and X in ohms.